

# Does a Kalb–Ramond field make spacetime optically active ?

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A spacetime with torsion produced by a Kalb-Ramond field coupled gravitationally to the Maxwell field, in accordance with a recent proposal by two of us (PM and SS), is argued to lead to an optical activity in synchrotron radiation from cosmologically distant radio sources. We suggest that this could *qualitatively* explain observational data from a large number of radio sources displaying such polarization asymmetry (after eliminating effects of Faraday rotation due to magnetized galactic plasma). Possible implications for heterotic string theory are also outlined.

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## I. INTRODUCTION

The massless antisymmetric tensor Kalb-Ramond (KR) field has been an inherent aspect of supergravity theories. Indeed, the tensor multiplet in  $N = 1$  theories has interesting duality properties that are exploited in its coupling to supergravity [1]. In extended supergravity, the KR field becomes a part of the supergravity multiplet itself, thus playing a more intrinsic role. The importance of the KR field in supergravity theories in various spacetime dimensions has been emphasized more than ever in string theories [2]. Supergravity multiplets constitute the massless sector of string theories. As such, they inevitably contain massless KR fields. Such fields implement a spacetime background for string theory possessing *torsion* in addition to curvature.

One aspect which particularly deserves attention in this respect is that of cosmology in the presence of torsion, or, equivalently, in the presence of a KR field. Indeed, the cosmological domain is the most likely arena for physical ‘stringy’ effects to appear. Since sources of torsion exist in the massless spectra of most viable string theories (in the form of the KR field), at least in the perturbative sector, cosmological models with non-zero torsion need to be investigated. Restricting ones attention to Einstein-Cartan spacetimes, the issue of gauge invariant coupling to standard massless gauge fields arises. The well-known problem [3] associated with the Maxwell field has been addressed in [4] by introducing a KR field, and augmenting it in accord with requirements of quantum consistency of heterotic string theory toroidally compactified to four spacetime dimensions.

In this paper, we examine the consequences of the resulting dynamics, to discern effects that could, even if remotely, be astrophysically/cosmologically observable. The possibility that a KR field may induce a rotation of the plane of polarization of electromagnetic radiation from cosmologically distant sources, was already alluded to in [4]. There is some evidence that optical activity of a related type may have already been *observed* in radiation from distant quasars and other radio sources [5].<sup>1</sup> Typically, the observed angle of rotation of the plane of polarization can be expressed as [7] :

$$\theta = \alpha\lambda^2 + \chi \quad (1)$$

where  $\alpha$  (the Faraday rotation measure) and  $\chi$  are constants and  $\lambda$  is the wavelength of the electromagnetic wave.  $\chi$  is the angle between a reference axis and the electric field of the wave when it is emitted from the source galaxy, while the first term, by dint of its dependence on the wavelength quadratically, represents *Faraday* rotation of the plane

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<sup>1</sup>Earlier analyses of the data on this topic [7] (and the data itself) were infused with a certain amount of controversy [8]; later works appear to be free of most of the contentious issues [6].

of polarization due to passage of the electromagnetic wave through galactic (and possibly inter-galactic) magnetized plasmas. The key aim of this paper is to argue that Einstein-Kalb-Ramond-Maxwell coupling can be responsible, albeit qualitatively, in explaining the origin of this extra bit of rotation ( $\chi$ ). More precisely, the question is whether the observed values of  $\chi$  can be explained by *assuming* that the plane of polarisation of the wave emitted is initially along a fixed angle (0 or  $\frac{\pi}{2}$ , in accordance with the source models for elliptical galaxies and the nature of synchrotron radiation) relative to the galaxy major axis (note that the major axis is tilted w.r.t. the reference axis by an angle  $\psi$  known as the intrinsic position angle of the corresponding galaxy), but undergoes a rotation due to the presence of the KR field. We recall that there is no extant proposal based on fundamental physics to account for  $\chi$ .

A result in support of this proposal is most likely an evidence of the existence of a primordial KR field and hence of a spacetime with (non-propagating) torsion in an early epoch. Since a KR field does occur naturally in some supergravity theories and hence in the massless spectrum of closed string theory, such an observation may perhaps be construed to be evidence for supergravity as well as being a hint of an underlying string structure.

A remark on our basic strategy is perhaps in order: in the sequel, we treat the KR field as a *tiny* perturbation on the Maxwell field equations in a standard cosmological background. In other words, despite the assumed ‘primordial’ origin of the KR field, we restrict its strength to such small values that its energy density plays an insignificant role in shaping geometry on a cosmological scale. For the latter, we choose two standard scenarios, viz., the *spatially* flat Friedmann-Robertson-Walker background with the scale factor, which depends only on (comoving) time, evolving according to both a radiation dominated and a matter dominated scheme. This is, of course, an approximation which we hope to improve in future assays on this subject. It is primarily motivated by the fact that there may not exist exact solutions of the KR-coupled Einstein equation for the evolution of spacetime geometry, which are also homogeneous *and* isotropic. There is no compelling observational evidence yet to doubt these latter requirements.

The paper is organized as follows: in Section II, we review the important aspects of the earlier paper [4] as background for the present work. This is followed in Section III by a presentation of the solution of Maxwell-KR field equations in a flat background spacetime and retaining only the leading order coupling of the two fields. The first hint of an optical activity already appears at this preliminary stage. Sections IV constitutes the main part of the work, where, in a spatially flat FRW background, the Maxwell-KR equations are considered for conformal factors pertaining to the radiation and matter dominated scenarios. Expressions for the angle of rotation of the plane of polarization are obtained, as a function of the redshift, for very large co-moving times, both in the radiation and matter dominated cosmological settings. We conclude in Section V.

## II. EINSTEIN-MAXWELL-KALB-RAMOND COUPLING

Let us briefly recapitulate the main tenets of the earlier paper [4]. It is well known that the electromagnetic field tensor, defined as the generally covariant curl of the four potential, is not invariant under the standard  $U(1)$  electromagnetic gauge transformation  $\delta A_\mu = \partial_\mu \omega$ , assuming that the torsion tensor  $T_{\mu\nu}^\rho$  - a purely geometric quantity like curvature must be gauge invariant. We introduce a Kalb-Ramond (KR) antisymmetric second rank tensor field  $B_{\mu\nu}$  as a possible source of torsion. The KR field strength is modified by  $U(1)$  Chern-Simons terms which originates from the quantum consistency of an underlying string theory [2]. This augmented field is coupled to the torsion in a way that the resulting action preserves all gauge symmetries and has the form

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R(g, T) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \tilde{H}_{\mu\nu\lambda} \tilde{H}^{\mu\nu\lambda} + \frac{1}{\sqrt{G}} T^{\mu\nu\lambda} \tilde{H}_{\mu\nu\lambda} \right] \quad (2)$$

where  $R$  is the scalar curvature, defined as  $R = R_{\alpha\mu\beta\nu} g^{\alpha\beta} g^{\mu\nu}$ .  $R_{\alpha\mu\beta\nu}$  is the Riemann-Christoffel tensor:

$$R_{\mu\nu\lambda}^\kappa = \partial_\mu \Gamma_{\nu\lambda}^\kappa - \partial_\nu \Gamma_{\mu\lambda}^\kappa + \Gamma_{\mu\sigma}^\kappa \Gamma_{\nu\lambda}^\sigma - \Gamma_{\nu\sigma}^\kappa \Gamma_{\mu\lambda}^\sigma \quad (3)$$

The torsion tensor  $T_{\mu\nu\lambda}$  is an auxiliary field in eq. (2), obeying the constraint equation

$$T_{\mu\nu\lambda} = \sqrt{G} \tilde{H}_{\mu\nu\lambda}, \quad (4)$$

where,  $\tilde{H}_{\mu\nu\lambda} \equiv \partial_{[\mu} B_{\nu\lambda]} + \frac{1}{3} \sqrt{G} A_{[\mu} F_{\nu\lambda]}$  [4]. Thus, the augmented KR field strength three tensor plays the role of the spin angular momentum density which is the source of torsion [9]. Substituting the above equation in (2) and varying with respect to  $B_{\mu\nu}$  and  $A_\mu$  respectively, we obtain the equations

$$D_\mu \tilde{H}^{\mu\nu\lambda} = 0 \quad (5)$$

and

$$D_\mu F^{\mu\nu} = \sqrt{G} \tilde{H}^{\mu\nu\lambda} F_{\lambda\mu} . \quad (6)$$

Now, the KR three tensor is Hodge-dual to the derivative of a spinless field  $\phi$ , so that, after a partial integration, one obtains,

$$S_{int} = \frac{1}{2} \int d^4x \phi F_{\mu\nu} {}^*F^{\mu\nu} , \quad (7)$$

where,  ${}^*F^{\mu\nu} \equiv \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma}$ . Here, we have noted the fact that

$$\frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} {}^*F^{\mu\nu}) = D_\mu^S {}^*F^{\mu\nu} = 0 \quad (8)$$

by the Maxwell Bianchi identity, where,  $D^S$  is the covariant derivative using the Christoffel connection.

In general, in addition to the graviton and the KR field, the perturbative sector of the heterotic string contains a scalar dilaton field whose dynamics is also known to have cosmological consequences [10]. In this paper, we shall however ignore this dynamics for the moment and focus instead on what the KR field does. In any event, the dilaton field couples to the Maxwell Lagrangian and the kinetic term of the KR field, and so cannot affect in any major way the optical activity induced by the axion (KR) field; the latter effects appear due to the fact that  $F^*F$  is *pseudoscalar*. Freezing the dilaton field and taking the KR field strength to be

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho}^\sigma D_\sigma H ,$$

where  $H$  is a pseudoscalar, one obtains the modified generally covariant Maxwell equations [4]

$$\begin{aligned} \mathbf{D} \cdot \mathbf{E} &= 2\sqrt{G} \mathbf{D} H \cdot \mathbf{B} \\ D_0 \mathbf{E} - \mathbf{D} \times \mathbf{B} &= -2\sqrt{G} [D_0 H \mathbf{B} - \mathbf{D} H \times \mathbf{E}] \\ &\quad + 2G[(\mathbf{B}^2 - \mathbf{E}^2) \mathbf{A} + (\mathbf{A} \cdot \mathbf{E}) \mathbf{E} + (\mathbf{A} \cdot \mathbf{B}) \mathbf{B}] \\ D_0 \mathbf{B} + \mathbf{D} \times \mathbf{E} &= 0 = \mathbf{D} \cdot \mathbf{B} . \end{aligned} \quad (9)$$

Here  $D_\mu$  is the covariant derivative in the spatially flat FRW metric. To a first approximation, we drop the  $O(G)$  terms arising from the stringy augmentation of the KR field strength in terms of the Chern Simons three form. Next, redefine the pseudoscalar field  $H$  to absorb the  $\sqrt{G}$ , so that this field becomes dimensionless. The equations now look like

$$\begin{aligned} \mathbf{D} \cdot \mathbf{E} &= 2 \mathbf{D} H \cdot \mathbf{B} \\ D_0 \mathbf{E} - \mathbf{D} \times \mathbf{B} &= -2 [D_0 H \mathbf{B} - \mathbf{D} H \times \mathbf{E}] \\ D_0 \mathbf{B} + \mathbf{D} \times \mathbf{E} &= 0 = \mathbf{D} \cdot \mathbf{B} . \end{aligned} \quad (10)$$

The last equations in the array constitute the Maxwell Bianchi identity. In a spatially flat isotropic FRW background with metric

$$ds^2 = R^2(\eta) (d\eta^2 - d\mathbf{x}^2) , \quad (11)$$

where,  $\eta$  is the conformal time coordinate, defined by  $d\eta = dt/R(t)$ , the above equations assume the form,

$$\begin{aligned} \nabla \cdot \mathbf{E} R^2 &= 2 \nabla H \cdot \mathbf{B} R^2 \\ \partial_\eta (\mathbf{E} R^2) - \nabla \times \mathbf{B} R^2 &= -2 [\partial_\eta H \mathbf{B} R^2 - \nabla H \times \mathbf{E} R^2] \\ \partial_\eta (\mathbf{B} R^2) + \nabla \times \mathbf{E} R^2 &= 0 = \nabla \cdot \mathbf{B} R^2 . \end{aligned} \quad (12)$$

### III. FLAT UNIVERSE

We first consider the simple situation corresponding to a *flat* background spacetime ( $R(\eta) = 1$ ), just to obtain a preliminary understanding of the effects involved. This simplification does not in any way reduce the qualitative

aspects of the optical effects under discussion, although the quantitative details obtained in this manner may not be reliable. Recall that the KR field strength  $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$ , so that it satisfies the Bianchi identity

$$\epsilon^{\mu\nu\lambda\sigma} \partial_\sigma H_{\mu\nu\lambda} = 0. \quad (13)$$

This immediately implies that the pseudoscalar  $H$  satisfies the massless Klein-Gordon eqn  $\square H = 0$ . For non-flat backgrounds, the d' Alembertian operator is to be replaced by its generally covariant counterpart. We note that this is a departure from approaches in the literature where the axion (KR) field  $H$  is introduced ad hoc with no specified dynamics; here the Bianchi identity of the dual KR field is precisely the equation of motion of the axion. Assume now that  $H$  is only a function of the comoving time coordinate  $\eta$ , so that, the Klein-Gordon equation reduces to the simple equation  $\frac{d^2 H}{d\eta^2} = 0$  with the obvious solution  $H = h\eta + h_0$ , where  $h$  and  $h_0$  are constants. This spatial homogeneity of the Klein-Gordon field is possibly a justified assumption over the cosmologically long distance scales of our interest.

Proceeding along the lines of [11] and [12], we arrive at the equation

$$\frac{d^2 b_\pm}{d\eta^2} + (k^2 \mp 2h) b_\pm = 0, \quad (14)$$

where we have decomposed  $\mathbf{B} = \mathbf{b}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}$  and have chosen the  $z$  direction to be the propagation direction of the electromagnetic wave. The circular polarization states are defined as  $b_\pm \equiv b_x \pm ib_y$ . Unlike the corresponding equation in [11], eqn (14) can be solved *exactly* :

$$b_\pm = b_0 e^{i\omega_\pm \eta} \equiv b_0 e^{i\phi_\pm}, \quad (15)$$

where,  $\omega_\pm^2 \equiv k(k \mp 2h)$ . The optical activity due to the presence of the KR field is thus given by the difference

$$(\Delta\phi)_{mag} \equiv \frac{1}{2}(\phi_+ - \phi_-) = -h\eta \quad \text{for } k \gg h \quad (16)$$

We also note that the equation for the electric field (with an assumption  $\mathbf{E} = \mathbf{e}(\eta)e^{ikz}$  and a similar definition for  $e_\pm$ ) takes the form :

$$\frac{d^2 e_\pm}{d\eta^2} + k^2 e_\pm = -2h \frac{db_\pm}{d\eta} \quad (17)$$

A solution of the electric field equation is therefore dependent on the solution of the magnetic field equation. It is easy to see that,

$$e_\pm = \mp \frac{b_0}{k} \omega_\pm i e^{i\omega_\pm \eta} \quad (18)$$

is a solution for the electric equation and the amount of rotation is the same for both the electric and magnetic fields.

All this is clear indication of an optical activity induced by the KR field. It is quite unlikely that this effect will disappear when cosmological backgrounds with curvature, the dilaton field or indeed the Chern Simons terms are included in the Maxwell equations; these features must be incorporated before a detailed comparison with any observational result can be made.

#### IV. SPATIALLY FLAT FRW UNIVERSE: RADIATION AND MATTER DOMINATED CASES

The next immediate step in our analysis is to solve Maxwell equations once again, but in a non-trivial cosmology - we choose for simplicity the *spatially* flat Friedman–Robertson–Walker (FRW) type of background.

The equation of motion of the pseudoscalar field is given as :

$$\square H = 0 \quad (19)$$

where  $\square$  is now the covariant d'Alembertian appropriate to the spatially flat metric.

For a spatially independent  $H$  field, such that  $H \equiv H(\eta)$  (19) has a first integral of the form

$$\partial_0 H = \frac{h}{R^2(\eta)} \quad (20)$$

where  $h$  is an integration constant, which, in a sense, is a ‘measure’ of the pseudoscalar  $H$  field or, equivalently, the dual three form field  $H_{\mu\nu\lambda}$ .

The equations that the polarization states  $b_{\pm}$  satisfy for such a background can be similarly written down in terms of the quantity  $F_{\pm}$  where  $b_{\pm} = \frac{F_{\pm}}{R^2}$ . They are

$$\frac{d^2 F_{\pm}}{d\eta^2} + (k^2 \mp \frac{2 h k}{R^2(\eta)}) F_{\pm} = 0 , \quad (21)$$

As for the flat spacetime case, a corresponding equation for the electric field polarisation states  $e_{\pm}$  can also be obtained in terms of a quantity  $G_{\pm}$  where  $e_{\pm} = \frac{G_{\pm}}{R^2}$ . This turns out to be :

$$\left( \frac{d^2}{d\eta^2} + k^2 \right) G_{\pm} = -2h \frac{d}{d\eta} \left[ \frac{F_{\pm}}{R^2} \right] \quad (22)$$

Note, that the electric field equations are dependent on the solution of their magnetic field counterparts. The magnetic field equations, however, can be solved without any reference to the electric ones. The rotation can indeed be calculated for both the electric and magnetic fields and we demonstrate our results for the magnetic field case in the sections below.

The equations, of course, can be solved explicitly only after a knowledge of the scale factor  $R(\eta)$  is available. One may resort to a WKB approximation along the lines of [11] and arrive at qualitative results. We prefer, as an alternative exercise, to choose a ‘physically reasonable’ scale factor and derive the effects of optical activity for such a case. As mentioned earlier, the actual situation will correspond to a scale factor corresponding to a solution of the Einstein-KR-Maxwell-dilaton equations of motion for low energy effective supergravity. However, since the dilaton couples to the Maxwell Lagrange density and not to  $F * F$ , it is unlikely that the effect that we find without it will be washed away by its inclusion.

Let us assume a scale factor  $R(\eta) = \eta/\eta_0^R$  which is equivalent, in real time, to the scale factor of a radiation dominated FRW model, for  $1/\eta_0^R = (8\pi G\epsilon_0/3)^{1/2}$ , with  $\epsilon_0$  being the primordial radiant energy density. For a matter dominated model we assume  $R(\eta) = (\eta/\eta_0^M)^2$ . Our objective is to obtain the *asymptotic* dependence on  $\eta$  and the parameters of the theory which are  $\tilde{h} = h(\eta_0^R)^2$ ,  $h' = h(\eta_0^M)^4$  and the wave number  $k$ .

Accordingly, we have the two equations for the radiation and matter dominated cases which we quote below.

$$\frac{d^2 F_{\pm}}{dx^2} + \left( 1 - \frac{\mu_{\pm}^2}{x^2} \right) F_{\pm} = 0 , \quad \mu_{\pm}^2 \equiv 2\tilde{h}k \quad (23)$$

and

$$\frac{d^2 F_{\pm}}{dx^2} + \left( 1 - \frac{\mu_{\pm}^2}{x^4} \right) F_{\pm} = 0 , \quad \mu_{\pm}^2 \equiv 2h'k^3 \quad (24)$$

In the above, we use dimensionless quantities throughout, with  $x = k\eta$

We now use the ansatz

$$F_{\pm}(x) = e^{ix} v_{\pm}(x) \quad (25)$$

so that, (23) and (24) reduce to

$$\frac{d^2 v_{\pm}}{dx^2} + 2i \frac{dv_{\pm}}{dx} - \frac{\mu_{\pm}^2}{x^2} v_{\pm} = 0 \quad (26)$$

$$\frac{d^2 v_{\pm}}{dx^2} + 2i \frac{dv_{\pm}}{dx} - \frac{\mu_{\pm}^2}{x^4} v_{\pm} = 0 \quad (27)$$

We are only interested in asymptotic solution of these equations for  $x \rightarrow +\infty$ . Accordingly, we choose a solution for both cases of the type

$$v_{\pm}(x) = v_0^{\pm} + \frac{v_1^{\pm}}{x} + \frac{v_2^{\pm}}{x^2} + \dots \quad (28)$$

This is an asymptotic solution, as given standard texts on differential equations. If you use this ansatz to calculate the various coefficients, you find that in both cases, *all* coefficients are proportional to  $v_0^{\pm}$ . For the radiation dominated

case, the coefficients are a finite series in powers of  $\tilde{h}$  ; for matter dominated, they are all proportional to  $h'$ . The answer for the angle of rotation of the polarization plane, to lowest non-trivial order in  $1/x$  (remember that  $x = k\eta$ ) is given by,

$$\Delta\phi = |\arg v_0^+ - \arg v_0^- + 2\tan^{-1}(\tilde{h}k/x)| \text{ for } RD \quad (29)$$

and

$$\Delta\phi = |\arg v_0^+ - \arg v_0^- + 2\tan^{-1}(h'k^3/3x^3)| \text{ for } MD \quad (30)$$

But, recalling that the angle of rotation must vanish in absence of our proposed interaction, we get  $\arg v_0^+ - \arg v_0^- = 0$ . Note that *no* assumption is made about the dependence of the coefficients on  $h$ . Thus, the answers for the angle of rotation are :

$$\Delta\phi = |2\tan^{-1}(\tilde{h}/\eta)| \text{ for } RD \quad (31)$$

and

$$\Delta\phi = |2\tan^{-1}(h'/3\eta^3)| \text{ for } MD \quad (32)$$

For very small  $h$ , the inverse tangent may be replaced by its argument. These are our predictions from theory (to the lowest order in  $h$ ) for the rotation angle, which may now be checked against the data. It is more convenient to rewrite the expressions in terms of the red-shift  $z$  – these, i.e. the expression for the lookback time can be obtained from the expression

$$t - t_0 = \frac{2}{3H_0} \left[ 1 - (1+z)^{-\frac{3}{2}} \right] \quad (33)$$

where  $H_0$  is the value of the Hubble parameter today. The relation between conformal time  $\eta$  and real time  $t$  can be easily obtained from the expression,  $a(\eta)d\eta = dt$ .

One can obtain expressions for the rotation of the electric field which to the lowest order turns out to be the same as for the magnetic field. We also note the fact that at the lowest order  $\mathbf{E} \cdot \mathbf{B}$  is equal to zero, but it may not be so beyond this order.

## V. CONCLUSIONS

A part of the rotation of the plane of polarisation of light emitted from distant galaxies obtained after subtracting out the Faraday component is claimed to be due to the presence of the Kalb–Ramond field. The modified Maxwell equations after the inclusion of the effects of the H field have been written and analysed both in flat and curved FRW backgrounds. The wave equations for the electric and magnetic fields are indeed different (unlike usual electromagnetism) and the effects on the rotation are generally different.

We have calculated the rotation for three different cases both for the electric and magnetic fields. The results are as follows.

(i) For a flat spacetime,  $(\Delta\phi)_{mag} = -h\eta$  (here  $\eta$  and the real time  $t$  are actually the same).  $(\Delta\phi)_{elec}$  may be the same or different. We are able to write down a solution for which the electric and magnetic field rotations are the same except for a phase of  $\frac{\pi}{2}$  between the two.

(ii) For the realistic scenarios, an asymptotic series solution valid in the large  $\eta$  regime is obtained for both the electric and magnetic fields. The rotation for the radiation and matter dominated cases go as  $\frac{1}{\eta}$  and  $\frac{1}{\eta^3}$  respectively. It is possible to use the expressions for conformal time in terms of the redshift and demonstrate the rotation explicitly in terms of the redshift.

In regard to astrophysical observations of the optical activity we have discussed, our primary motivation is the careful analysis by Jain and Ralston [5], which is free of contentious issues pertaining to the acquisition and analysis of data. Such issues were in focus after the incipient work of Nodland and Ralston [7], and appears to have been resolved satisfactorily. It is fair to say therefore, that there is definite evidence that the rotation of the plane of polarization of radiation travelling over cosmologically large distances is not entirely attributable to the Faraday rotation due to magnetic fields present in the galactic plasma. In this paper, we have presented arguments to the effect that the ‘primordial’ optical activity is quite likely due to a KR (axion) field which endows the spacetime in its immediate vicinity with torsion. It is perhaps not without significance that for both the radiation and matter

dominated scenarios, the calculated angle of rotation of the plane of polarisation is independent of wavelength. This property is shared by the angle  $\chi$  which is the intercept vide Eqn (1) of the straight line  $\theta$  versus  $\lambda^2$ .

All this is perhaps an indication that supergravity is at work, and what one is observing is perhaps a massless mode of an underlying string theory, hitherto unobserved because of its weak coupling to other matter. As pointed out in [13], such a weak coupling could be detected perhaps in future, if not through presently available data. We leave open the question whether the present data does actually substantiate our theoretical conclusions.

Apart from the astrophysical ramifications of our work, the fact that the only known proposal of coupling the Maxwell field to an Einstein-Cartan geometry in a gauge invariant manner leads directly to the optical activity discussed above can, in principle, be of significant use in the detection of torsion as a geometrical property of spacetime. More quantitative analysis of this aspect will form the subject of future publications.

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